Name:

Fall 2018 Math 245 Exam 2

Please read the following directions:
Please write legibly, with plenty of white space. Please fit your answers in the designated areas. Please print your name on the designated line, similarly to your quizzes (last name(s) in ALL CAPS). Writing your name incorrectly will cost you a point. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 1:00 and will end at 1:50; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Exam Total:	50		100
Quiz Ave:	50		100
Overall:	50		100

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. Proof by Contradiction theorem

b. Proof by Cases theorem

c. Proof by Induction

d. Proof by Reindexed Induction

Problem 2. Carefully define the following terms:

a. well-ordered

b. recurrence

c. big Omega

d. big Theta

Problem 3. Suppose that an algorithm has runtime specified by the recurrence relation $T_n = 2nT_{n/2} + 3$. Determine what, if anything, the Master Theorem tells us.

Problem 4. Use induction to prove that, for all $n \in \mathbb{N}$, $\frac{(2n)!}{n!n!} \geq 2^n$.

Problem 5. Let $a_n = n^{1.9} + n^2$. Prove that $a_n = O(n^2)$.

Problem 6. Let $x \in \mathbb{R}$. Prove that there is at most one $n \in \mathbb{Z}$ with $n - \frac{1}{2} \leq x < n + \frac{1}{2}$. Do not use any theorems about floors or ceilings.

Problem 7. Let $x \in \mathbb{R}$. Prove that there is at least one $n \in \mathbb{Z}$ with $n - \frac{1}{2} \leq x < n + \frac{1}{2}$. Do not use any theorems about floors or ceilings.

Problem 8. Solve the recurrence, with initial conditions $a_0 = 3, a_1 = 4$, and relation $a_n = 4a_{n-1} - 4a_{n-2}$ $(n \geq 2)$.

Problem 9. The Tribonacci numbers are given by initial conditions $T_0 = 0, T_1 = 1, T_2 = 1$, and recurrence relation $T_k = T_{k-1} + T_{k-2} + T_{k-3}$ $(k \geq 3)$. Prove that, for all $k \in \mathbb{N}$, $T_k < 2^k$.

Problem 10. Prove that $\sqrt{3}$ is irrational.